OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS

4726

Further Pure Mathematics 2

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:
None

Friday 22 May 2009
Morning

Duration: 1 hour 30 minutes

* 4 7 2 6 *

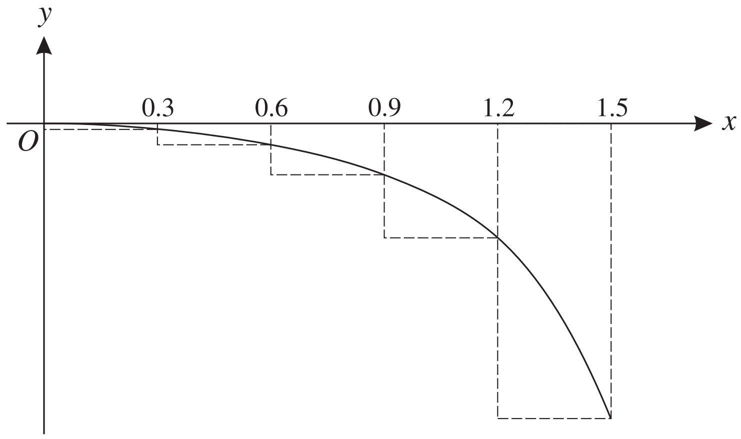
INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1



The diagram shows the curve with equation $y = \ln(\cos x)$, for $0 \leqslant x \leqslant 1.5$. The region bounded by the curve, the x-axis and the line $x = 1.5$ has area A. The region is divided into five strips, each of width 0.3.

(i) By considering the set of rectangles indicated in the diagram, find an upper bound for A. Give the answer correct to 3 decimal places. **[2]**

(ii) By considering another set of five suitable rectangles, find a lower bound for A. Give the answer correct to 3 decimal places. **[2]**

(iii) How could you reduce the difference between the upper and lower bounds for A? **[1]**

2 Given that $y = \dfrac{x^2 + x + 1}{(x-1)^2}$, prove that $y \geqslant \frac{1}{4}$ for all $x \neq 1$. **[4]**

3 **(i)** Given that $f(x) = e^{\sin x}$, find $f'(0)$ and $f''(0)$. **[4]**

(ii) Hence find the first three terms of the Maclaurin series for $f(x)$. **[2]**

4 Express $\dfrac{x^3}{(x-2)(x^2+4)}$ in partial fractions. **[6]**

5 It is given that $I = \displaystyle\int_0^{\frac{1}{2}\pi} \frac{\cos\theta}{1+\cos\theta}\,d\theta$.

(i) By using the substitution $t = \tan\frac{1}{2}\theta$, show that $I = \displaystyle\int_0^1 \left(\frac{2}{1+t^2} - 1\right) dt$. **[5]**

(ii) Hence find I in terms of π. **[2]**

6 Given that

$$\int_0^1 \frac{1}{\sqrt{16+9x^2}}\,dx + \int_0^2 \frac{1}{\sqrt{9+4x^2}}\,dx = \ln a,$$

find the exact value of a. **[6]**

7 **(i)** Sketch the graph of $y = \coth x$, and give the equations of any asymptotes. **[3]**

(ii) It is given that $f(x) = x\tanh x - 2$. Use the Newton-Raphson method, with a first approximation $x_1 = 2$, to find the next three approximations x_2, x_3 and x_4 to a root of $f(x) = 0$. Give the answers correct to 4 decimal places. **[4]**

(iii) If $f(x) = 0$, show that $\coth x = \frac{1}{2}x$. Hence write down the roots of $f(x) = 0$, correct to 4 decimal places. **[3]**

8 **(i)** Using the definitions of $\sinh x$ and $\cosh x$ in terms of e^x and e^{-x}, show that

(a) $\cosh(\ln a) \equiv \dfrac{a^2+1}{2a}$, where $a > 0$, **[3]**

(b) $\cosh x \cosh y - \sinh x \sinh y \equiv \cosh(x - y)$. **[3]**

(ii) Use part **(i)(b)** to show that $\cosh^2 x - \sinh^2 x \equiv 1$. **[1]**

(iii) Given that $R > 0$ and $a > 1$, find R and a such that

$$13\cosh x - 5\sinh x \equiv R\cosh(x - \ln a).$$ **[5]**

(iv) Hence write down the coordinates of the minimum point on the curve with equation $y = 13\cosh x - 5\sinh x$. **[2]**

9 **(i)** It is given that, for non-negative integers n,

$$I_n = \int_0^{\frac{1}{2}\pi} \sin^n\theta\,d\theta.$$

Show that, for $n \geqslant 2$,

$$nI_n = (n-1)I_{n-2}.$$ **[4]**

(ii) The equation of a curve, in polar coordinates, is

$$r = \sin^3\theta, \quad \text{for } 0 \leqslant \theta \leqslant \pi.$$

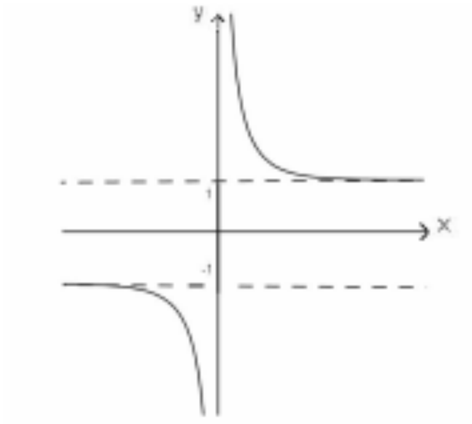
(a) Find the equations of the tangents at the pole and sketch the curve. **[4]**

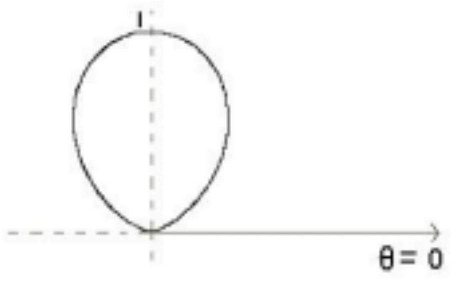
(b) Find the exact area of the region enclosed by the curve. **[6]**

4726 Further Pure Mathematics 2

1(i)	Attempt area = $\pm\Sigma(0.3y)$ for at least three y values	M1	May be implied
	Get 1.313(1..) or 1.314	A1	Or greater accuracy
(ii)	Attempt ± sum of areas (4 or 5 values)	M1	May be implied
	Get 0.518(4..)	A1	Or greater accuracy **SC** If answers only seen, 1.313(1..) or 1.314 B2 0.518(4..) B2 −1.313(1..) or −1.314 B1 −0.518(4..) B1
	Or Attempt answer to part **(i)**−final rectangle Get 0.518(4..)	 M1 A1	
(iii)	Decrease width of strips	B1	Use more strips or equivalent
2	Attempt to set up quadratic in x	M1	Must be quadratic; = 0 may be implied
	Get $x^2(y-1) - x(2y+1) + (y-1)=0$	A1	
	Use $b^2 \geq 4ac$ for real x on their quadratic	M1	Allow $=,>,<,\leq$ here; may be implied
	Clearly solve to AG	A1	If other (in)equalities used, the step to AG must be clear **SC** Reasonable attempt to diff. using prod/quot rule M1 Solve correct $dy/dx=0$ to get $x=-1$, $y=\frac{1}{4}$ A1 Attempt to justify inequality e.g. graph or to show $d^2y/dx^2>0$ M1 Clearly solve to AG A1
3(i)	Reasonable attempt at chain rule	M1	Product in answer
	Reasonable attempt at product/quotient rule	M1	Sum of two parts
	Correctly get $f'(0)=1$	A1	
	Correctly get $f''(0) = 1$	A1	**SC** Use of $\ln y = \sin x$ follows same scheme
(ii)	Reasonable attempt at Maclaurin with their values	M1	In $af(0) + bf'(0)x + cf''(0)x^2$
	Get $1 + x + \frac{1}{2}x^2$	A1√	From their f(0), f′(0), f″(0) in a correct Maclaurin; all non-zero terms
4	Attempt to divide out.	M1	Or $A+B/(x-2)+(Cx(+D))/(x^2+4)$; allow A=1 and/or B=1 quoted
	Get $x^3=$ $A(x-2)(x^2+4)+B(x^2+4)+(Cx+D)(x-2)$	M1	Allow √ mark from their Part Fract; allow D=0 but not C=0
	State/derive/quote A=1	A1	
	Use x values and/or equate coeff	M1	To potentially get all their constants

	Get B=1, C=1, D=-2	A1	For one other correct from cwo
		A1	For all correct from cwo
5(i)	Derive/quote $d\theta=2dt/(1+t^2)$	B1	May be implied
	Replace their cos θ and their $d\theta$, both in terms of t	M1	Not $d\theta = dt$
	Clearly get $\int(1-t^2)/(1+t^2)\,dt$ or equiv	A1	Accept limits of t quoted here
	Attempt to divide out	M1	Or use AG to get answer above
	Clearly get/derive AG	A1	
			SC Derive $d\theta = 2\cos^2\frac{1}{2}\theta\,dt$ B1 Replace $\cos\theta$ in terms of half-angles and their $d\theta$ ($\neq dt$) M1 Get $\int 2\cos^2\frac{1}{2}\theta - 1\,dt$ or $\int 1 - 1/2\cos^2\frac{1}{2}\theta\,.2/(1+t^2)\,dt$ A1 Use $\sec^2\frac{1}{2}\theta = 1+t^2$ M1 Clearly get/derive AG A1
(ii)	Integrate to $a\tan^{-1}bt - t$	M1	
	Get $\frac{1}{2}\pi - 1$	A1	
6	Get $k\sinh^{-1}k_1x$	M1	For either integral; allow attempt at ln version here
	Get $\frac{1}{3}\sinh^{-1}\frac{3}{4}x$	A1	Or ln version
	Get $\frac{1}{2}\sinh^{-1}\frac{2}{3}x$	A1	Or ln version
	Use limits in their answers	M1	
	Attempt to use correct ln laws to set up a solvable equation in a	M1	
	Get $a = 2^{\frac{1}{3}}.\,3^{\frac{1}{2}}$	A1	Or equivalent

7(i)		B1	y-axis asymptote; equation may be implied if clear
		B1	Shape
		B1	$y = \pm 1$ asymptotes; may be implied if seen as on graph
(ii)	Reasonable attempt at product rule, giving two terms	M1	
	Use correct Newton-Raphson at least once with their $f'(x)$ to produce an x_2	M1	May be implied
	Get $x_2 = 2.0651$	A1√	One correct at any stage if reasonable
	Get $x_3 = 2.0653$, $x_4 = 2.0653$	A1	cao; or greater accuracy which rounds
(iii)	Clearly derive coth $x = \frac{1}{2}x$	B1	AG; allow derivation from AG Two roots only
	Attempt to find second root e.g. symmetry	M1	
	Get $\pm$ 2.0653	A1√	$\pm$ their iteration in part **(ii)**
8(i)	**(a)** Get $\frac{1}{2}(e^{\ln a} + e^{-\ln a})$	M1	
	Use $e^{\ln a} = a$ and $e^{-\ln a} = {}^{1}/_{a}$	M1	
	Clearly derive AG	A1	
	(b) Reasonable attempt to multiply out their attempts at exponential definitions of cosh and sinh	M1	4 terms in each
	Correct expansion seen as $e^{(x+y)}$ etc.	A1	
	Clearly tidy to AG	A1	With $e^{-(x-y)}$ seen or implied
(ii)	Use $x = y$ and cosh0 = 1 to get AG	B1	
(iii)	Attempt to expand and equate coefficients	M1	($13 = R \cosh \ln a = R(a^2+1)/2a$ $5 = R \sinh \ln a = R(a^2-1)/2a$)
	Attempt to eliminate R (or a) to set up a solvable equation in a (or R)	M1	**SC** If exponential definitions used, $8e^{x} + 18e^{-x} = Re^{x}/a + Rae^{-x}$ and same scheme follows
	Get $a = {}^{3}/_{2}$ (or $R = 12$)	A1	
	Replace for a (or R) in relevant equation to set up solvable equation in R (or a)	M1	
	Get R=12 (or $a = {}^{3}/_{2}$)	A1	Ignore if $a = {}^{2}/_{3}$ also given
(iv)	Quote/derive ($\ln {}^{3}/_{2}$, 12)	B1√ B1√	On their R and a
9(i)	Use $\sin\theta.\sin^{n-1}\theta$ and parts	M1	Reasonable attempt with 2 parts, one yet to be integrated

	Get $-\cos\theta.\sin^{n-1}\theta+(n-1)\int\sin^{n-2}\theta.\cos^2\theta\ d\theta$	A1	Signs need to be carefully considered
	Replace $\cos^2 = 1 - \sin^2$	M1	
	Clearly use limits and get AG	A1	
(ii)	**(a)** Solve for r=0 for at least one θ	M1	θ need not be correct
	Get (θ) = 0 and π	A1	Ignore extra answers out of range
	 	B1	General shape (symmetry stated or approximately seen)
		B1	Tangents at θ=0, π and max r seen
	(b) Correct formula used; correct r	M1	May be $\int r^2\ d\theta$ with correct limits
	Use $6I_6 = 5I_4$, $4I_4 = 3I_2$	M1	At least one
	Attempt I_0 (or I_2)	M1	($I_0 = \frac{1}{2}\pi$)
	Replace their values to get I_6	M1	
	Get $5\pi/32$	A1	
	Use symmetry to get $5\pi/32$	A1	May be implied but correct use of limits must be given somewhere in answer
	Or		
	Correct formula used; correct r	M1	
	Reasonable attempt at formula $(2i\sin\theta)^6 = (z - {}^1/z)^6$	M1	
	Attempt to multiply out both sides (7 terms)	M1	
	Get correct expansion	A1	
	Convert to trig. equivalent and integrate their expression	M1	cwo
	Get $5\pi/32$	A1	
	Or		
	Correct formula used; correct r	M1	
	Use double-angle formula and attempt to cube (4 terms)	M1	
	Get correct expression	A1	
	Reasonable attempt to put $\cos^2 2\theta$ into integrable form and integrate	M1	
	Reasonable attempt to integrate $\cos^3 2\theta$ as e.g. $\cos^2 2\theta.\cos 2\theta$	M1	cwo
	Get $5\pi/32$	A1	